

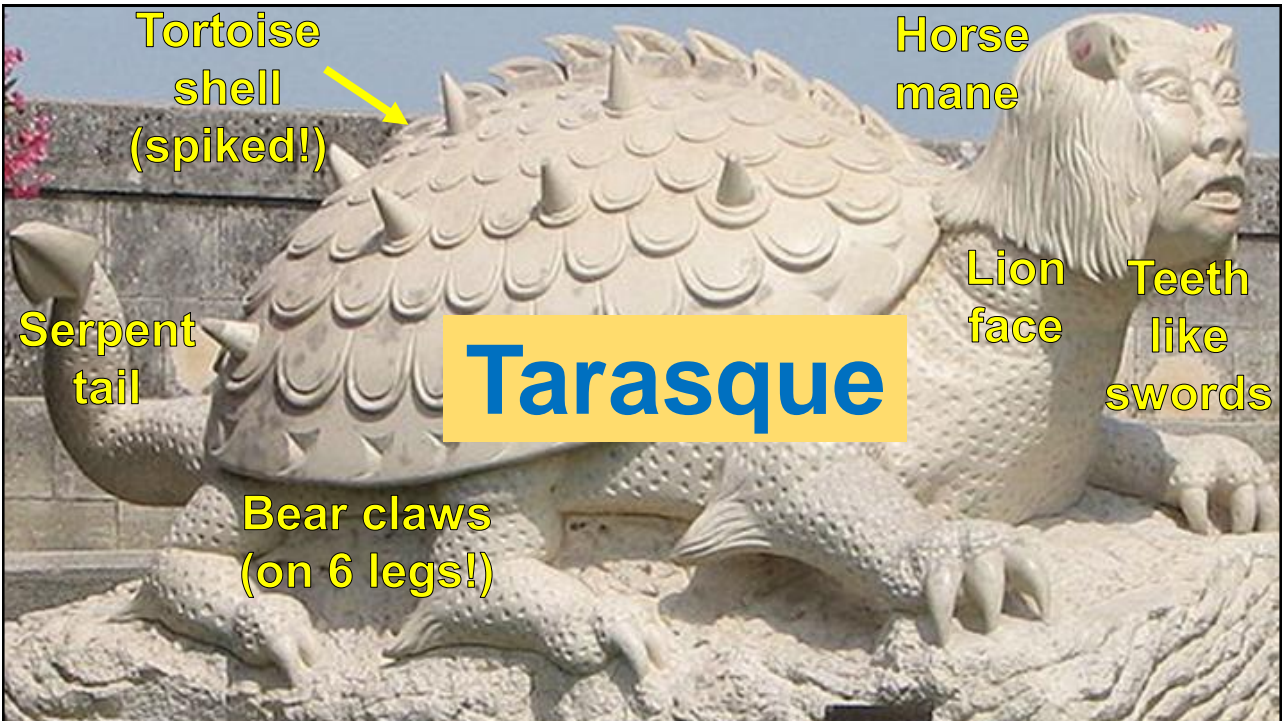
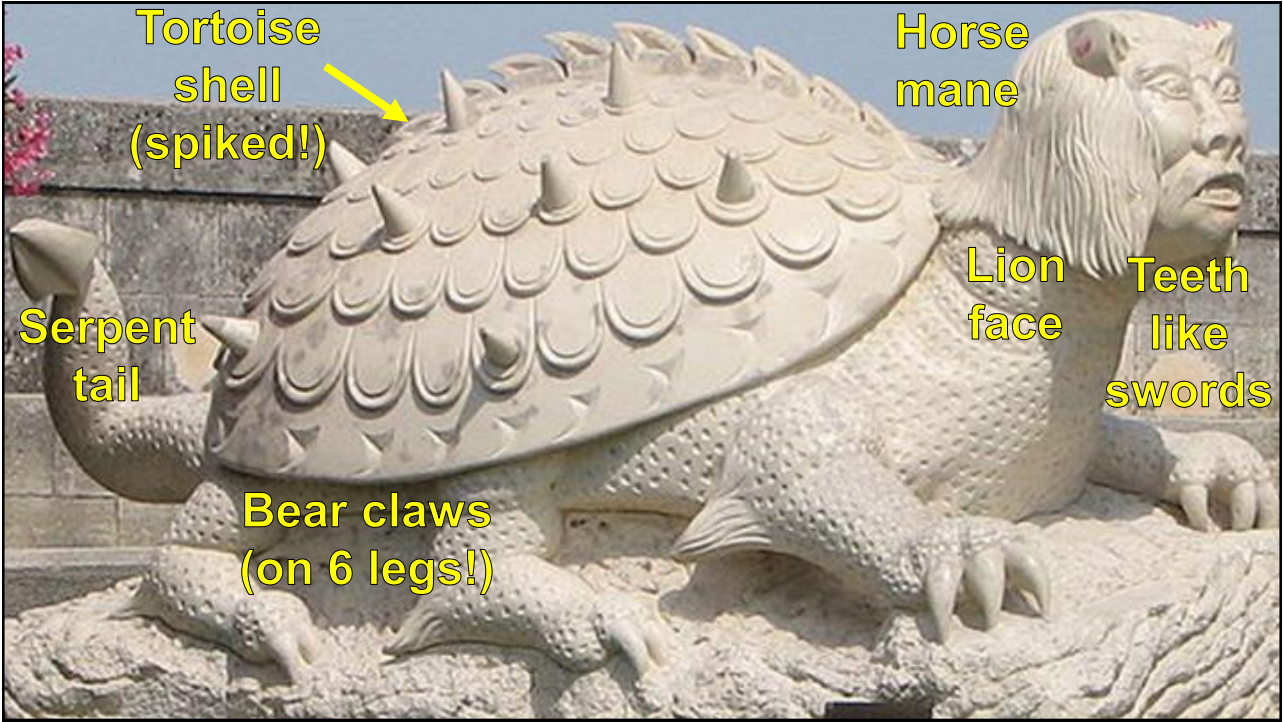


SOCIETY OF  
**SENSORY**  
PROFESSIONALS

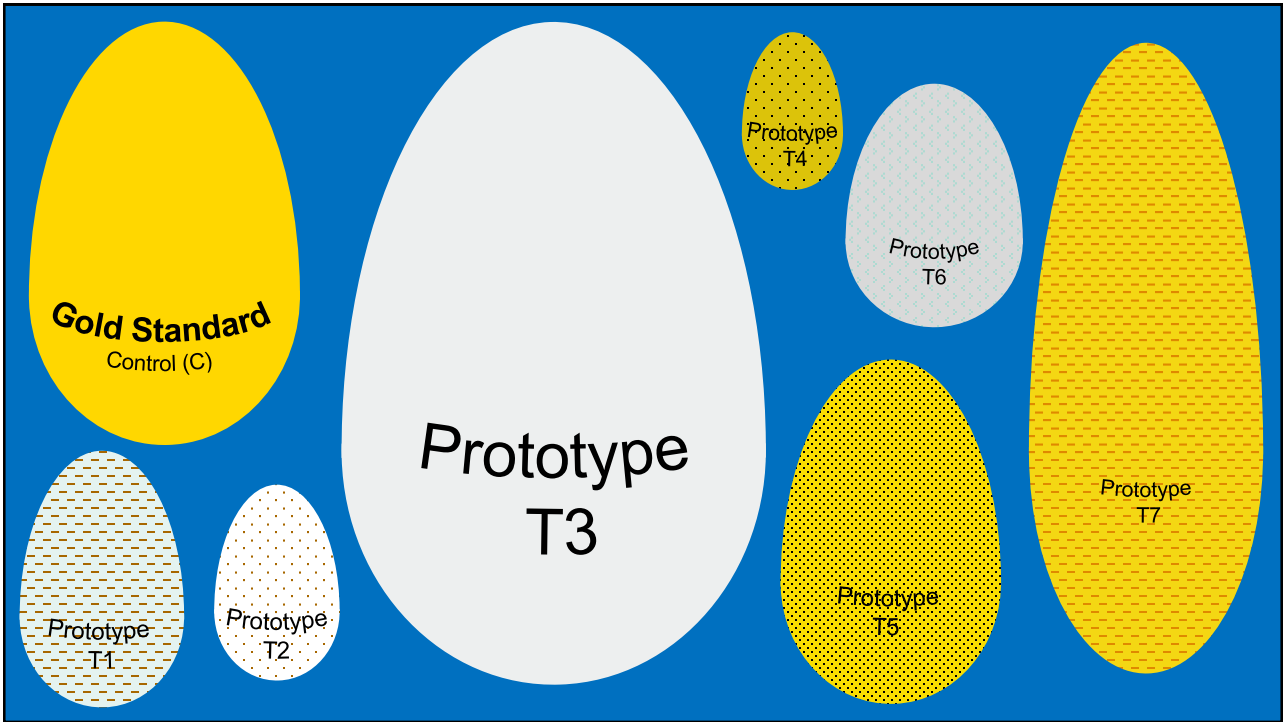
## Investigating relationships in sensory and instrumental data using component-based methods

**John C. Castura**  Compusense.



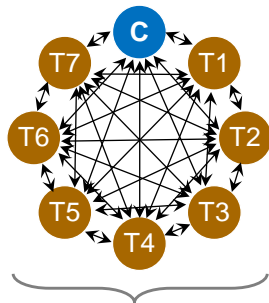






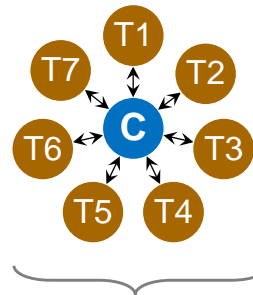
## All pairs vs. a subset of paired comparisons

### All Pairs



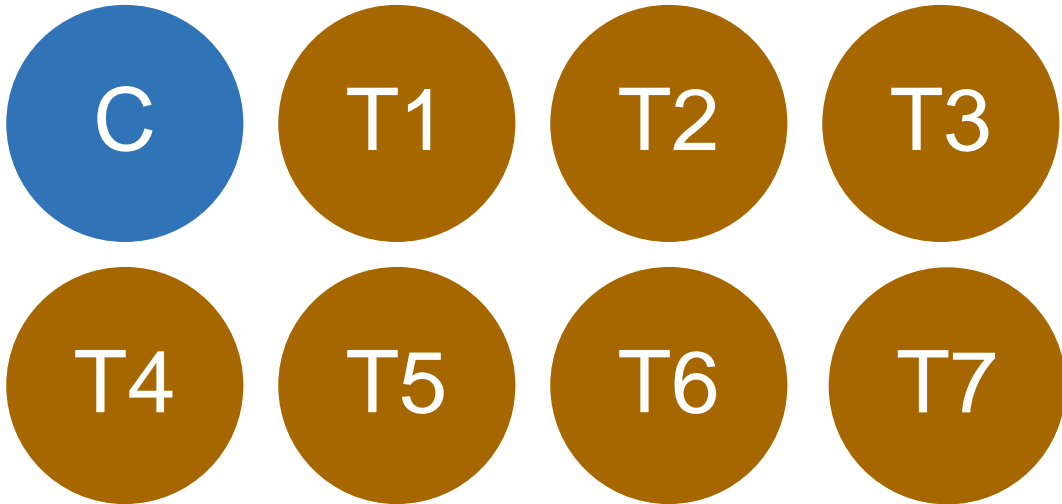
28 paired comparisons  
56 paired differences

### Test-Control Pairs



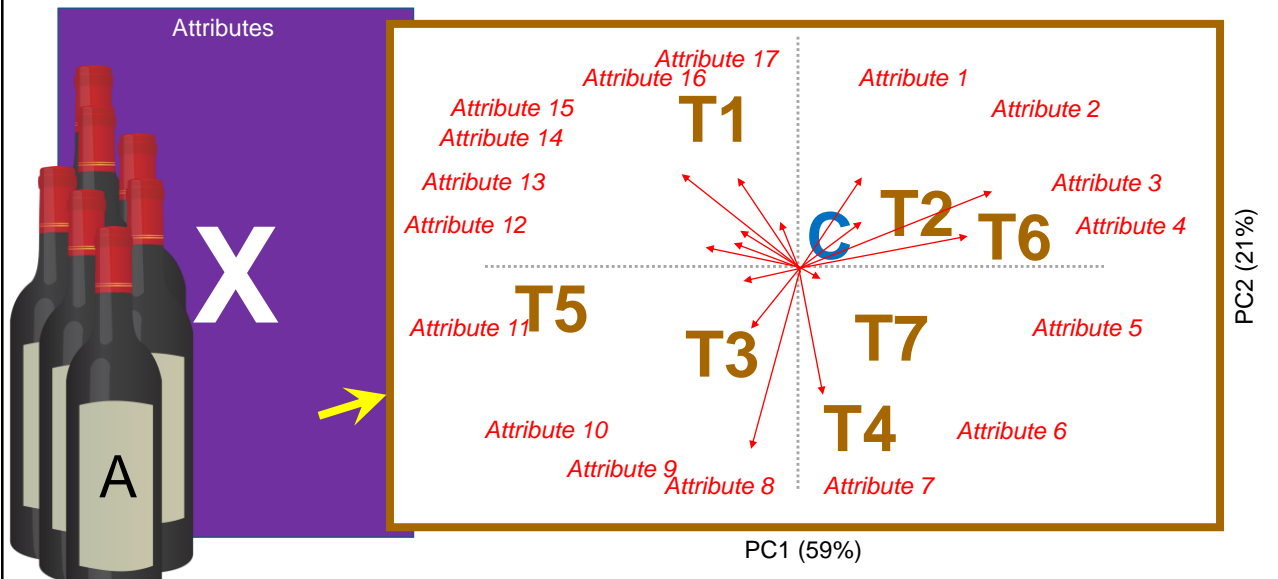
7 paired comparisons  
14 paired differences

Castura, J.C., Varela, P., & Næs, T. (2023). Investigating only a subset of paired comparisons after principal component analysis. *Food Quality and Preference*, 110, 104941. <https://doi.org/10.1016/j.foodqual.2023.104941>

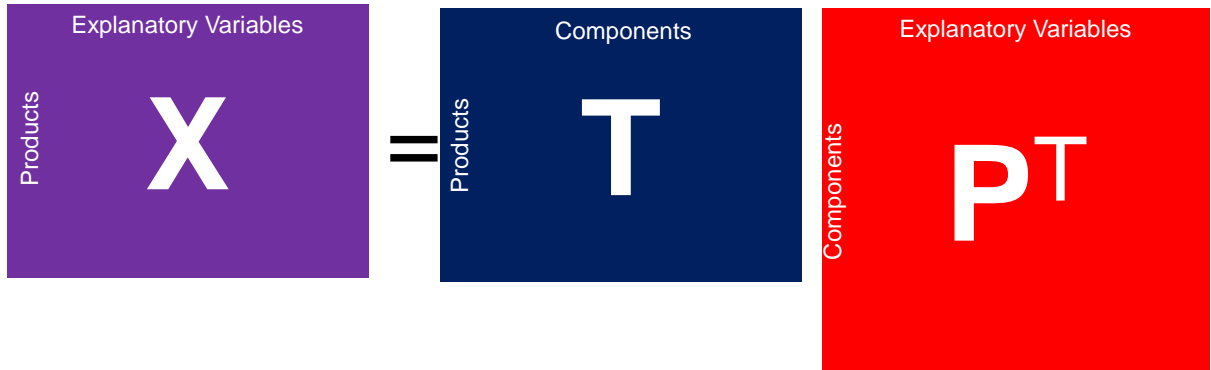


**Whatever shall we do?**

## Sensory evaluation

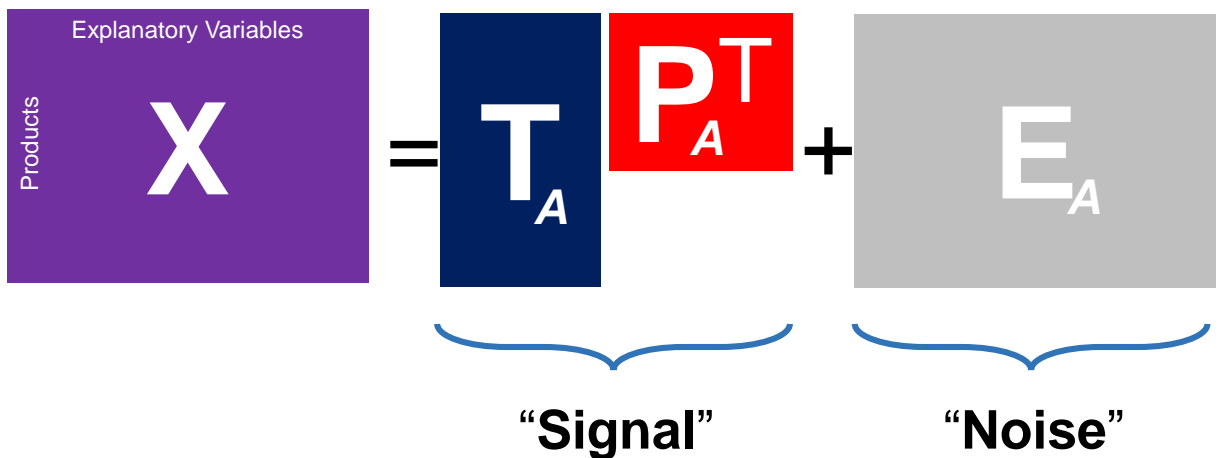


## Principal component analysis of X

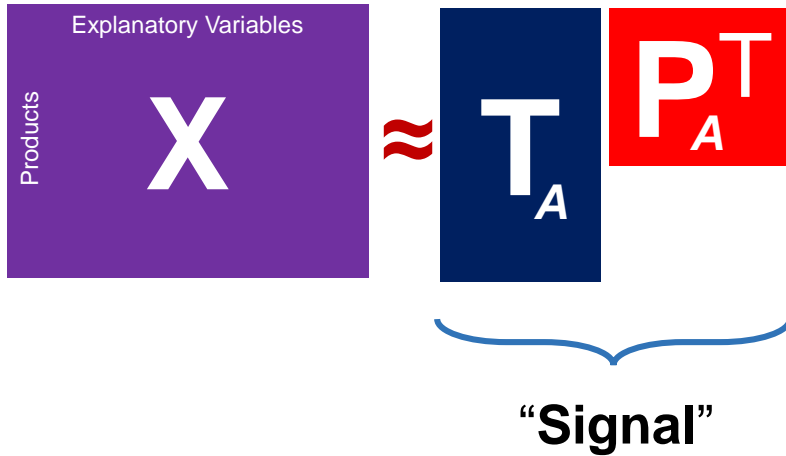


Pearson (1901) and Hotelling (1933)

## Dimension reduction to $A$ principal components

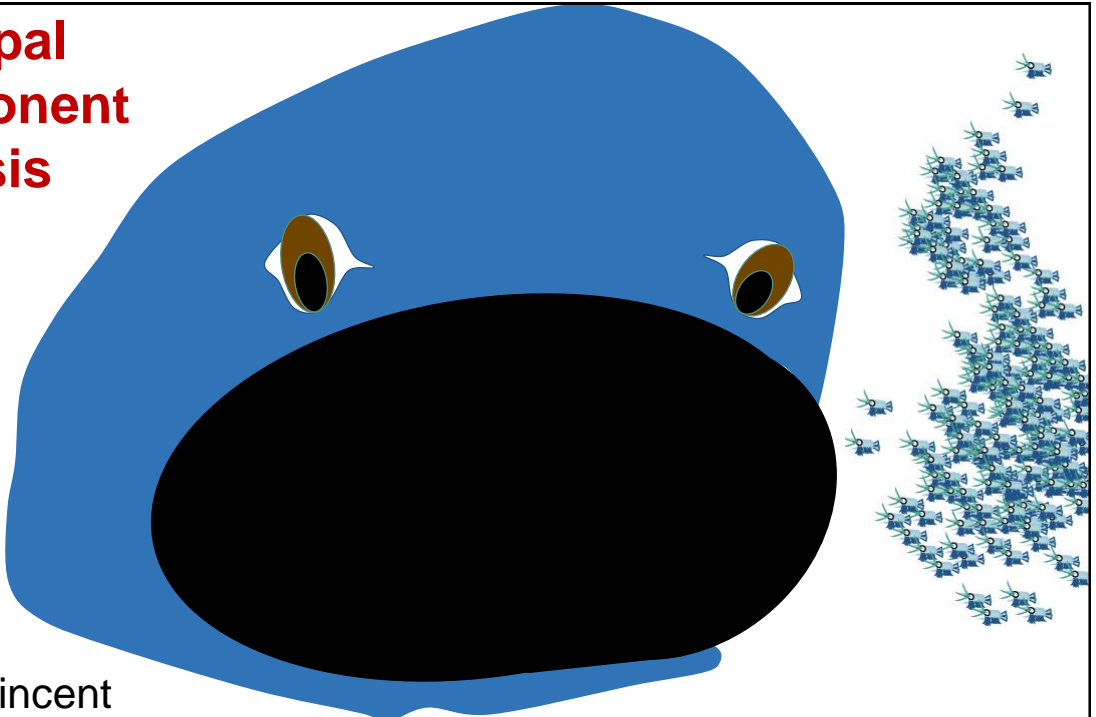


## Dimension reduction to $A$ principal components



## Principal component analysis

Baleen whale

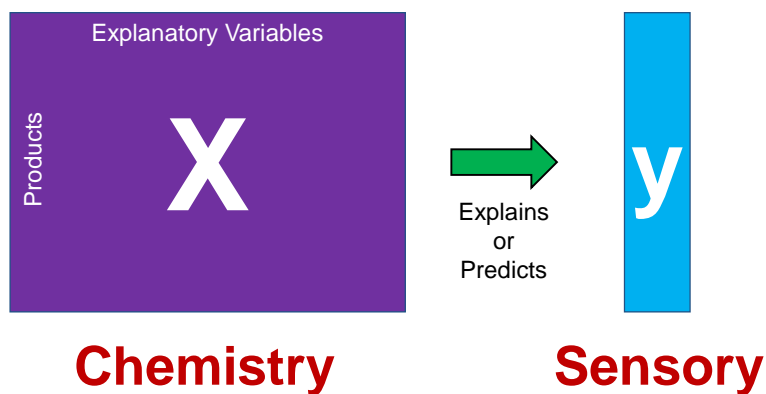


\*merci Vincent

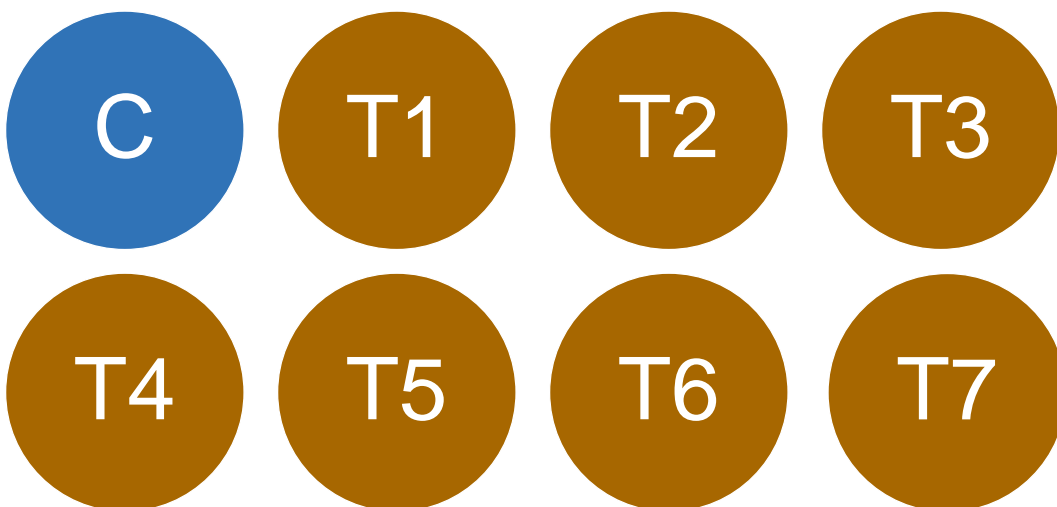
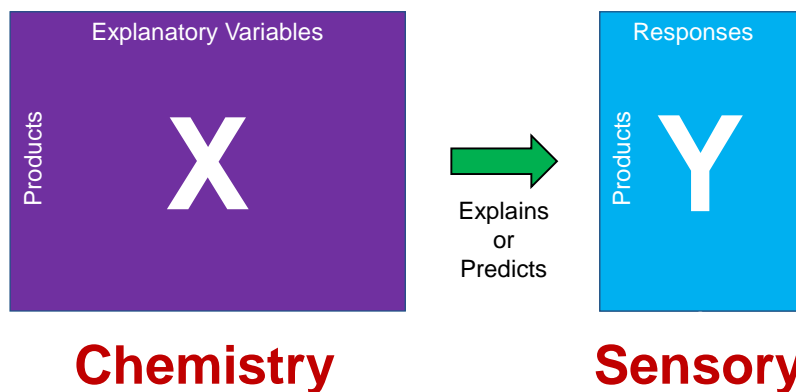
## Principal component



## Explaining and predicting data relationships



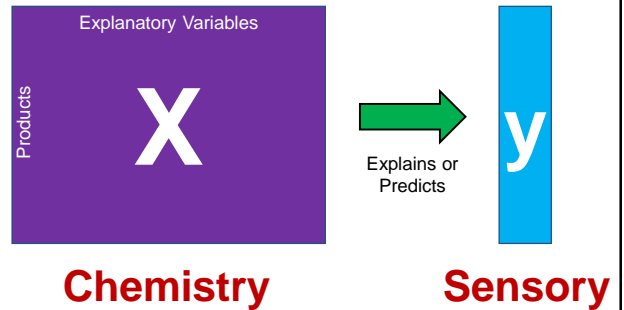
## Explaining and predicting data relationships



**Whatever shall we do?**







# Multiple linear regression

will not do the job

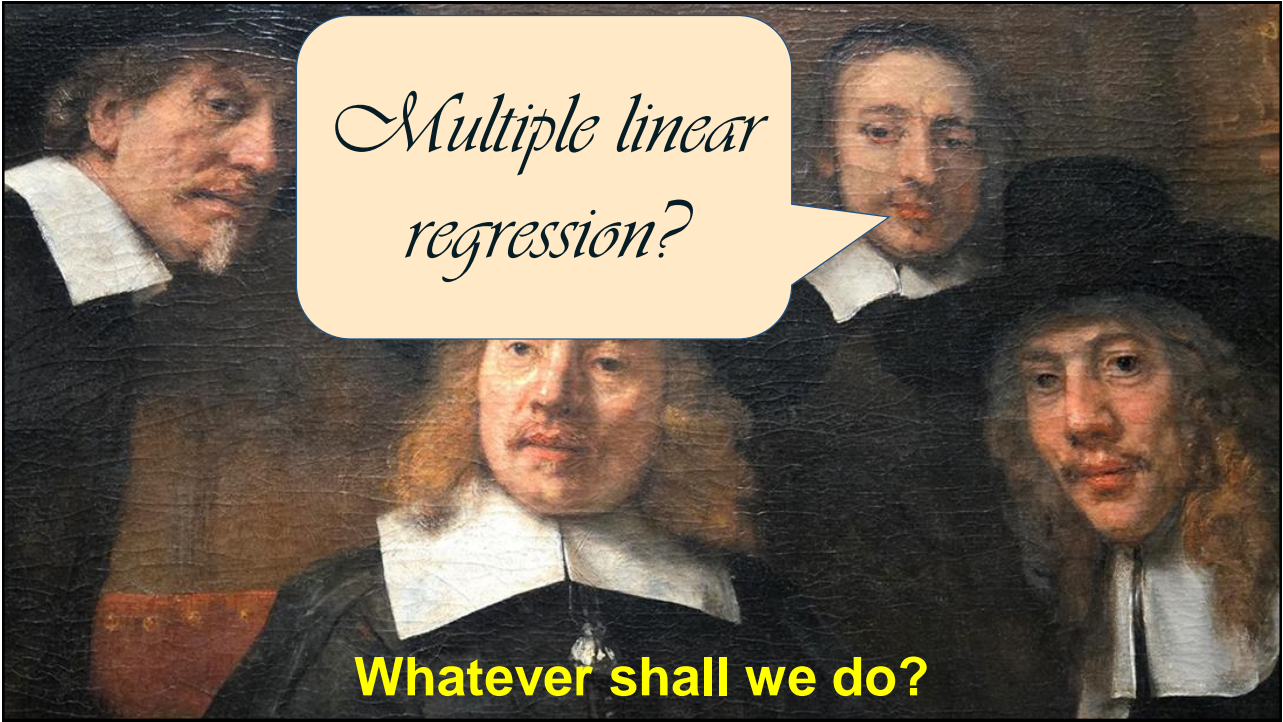
## Multiple Linear Regression

$$y = X\mathbf{b} + \mathbf{f}$$

Estimation problems due to...

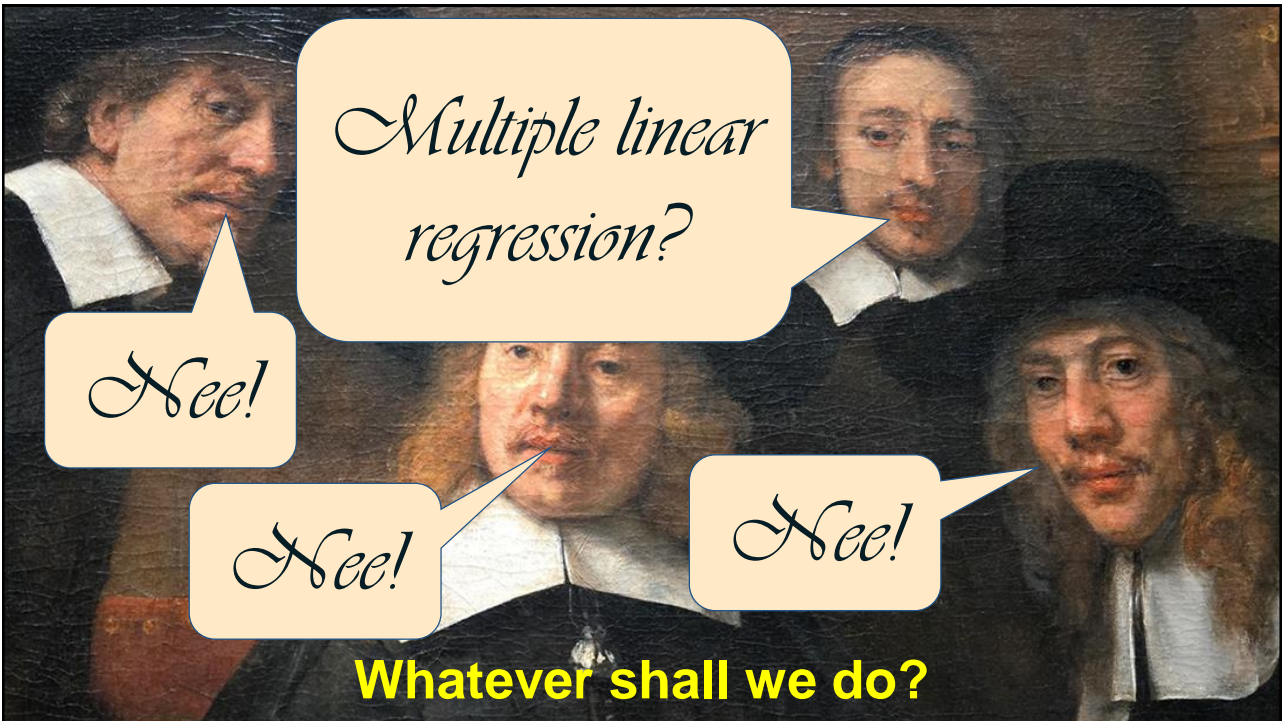
- multicollinearity
- more variables than objects





*Multiple linear regression?*

**Whatever shall we do?**



*Multiple linear regression?*

*Hee!*

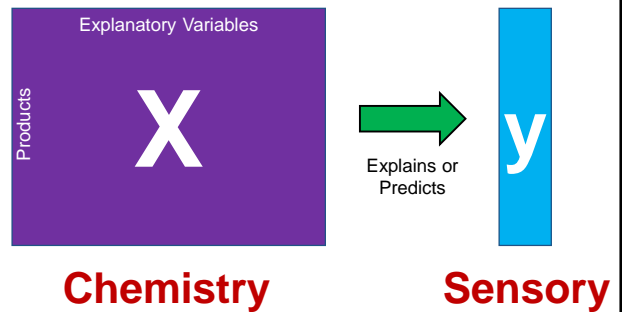
*Hee!*

*Hee!*

**Whatever shall we do?**







# Principal component regression

## Unsupervised & Supervised

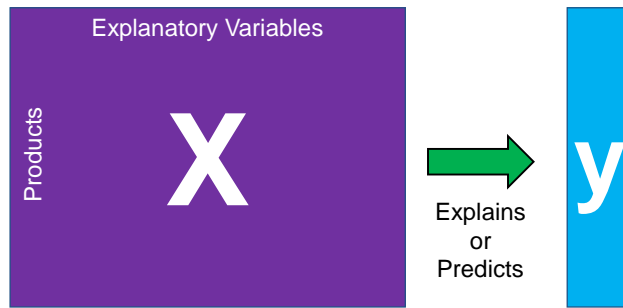
## Principal component regression (PCR)

We want to **explain** and **predict** the response **y** from multivariate **X**.

Response **y** is regressed on **principal components** of **X**.

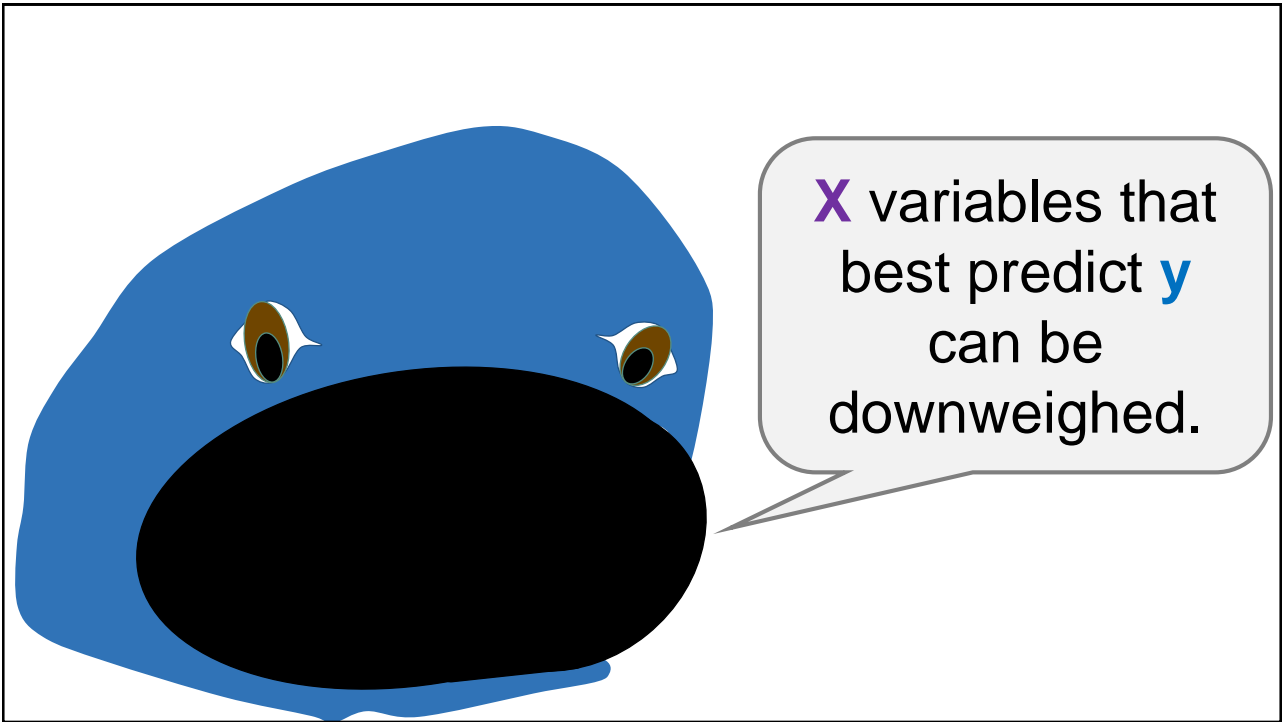


## Explaining and predicting data relationships



## Explaining and predicting data relationships





## Supervised principal component regression (SPCR)

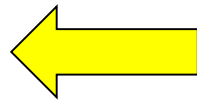
We want to **explain** and **predict** the response **y** from the multivariate **X**.

Response **y** is regressed on principal components of **X**. **SUPERVISED**

## Supervised principal component regression (SPCR)



Drop  $X$   
variables



that do  
not  
explain  $y$



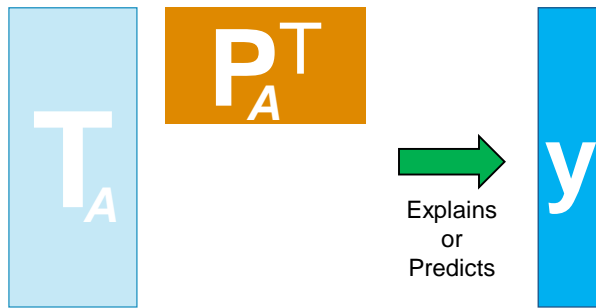
Bair, Hastie & Tibshirani (2006). Prediction by supervised principal components. *JASA*. [oi:10.1198/016214505000000628](https://doi.org/10.1198/016214505000000628)

## Dimension reduction in SPCR



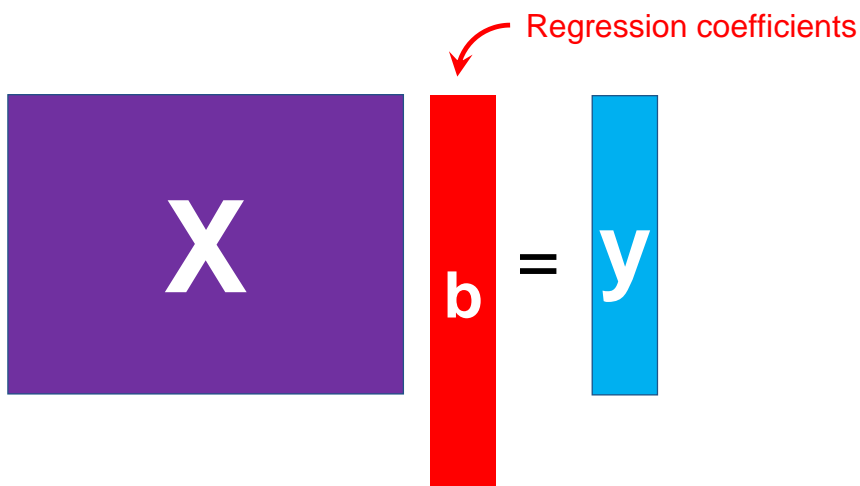
Bair, Hastie & Tibshirani (2006)

## Supervised principal component regression (SPCR)



Bair, Hastie &amp; Tibshirani (2006)

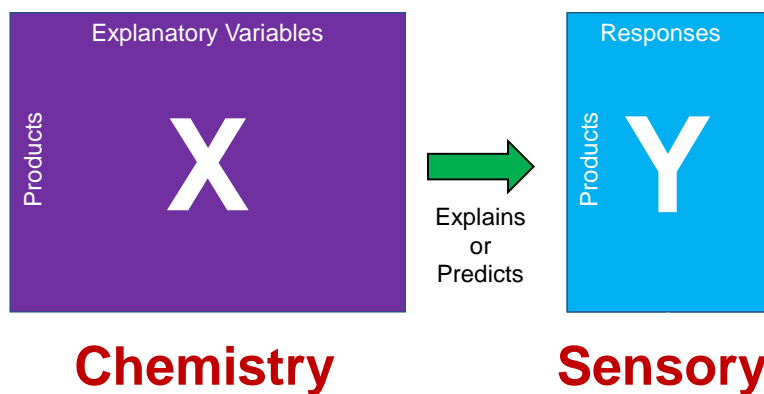
## Explaining and predicting data relationships



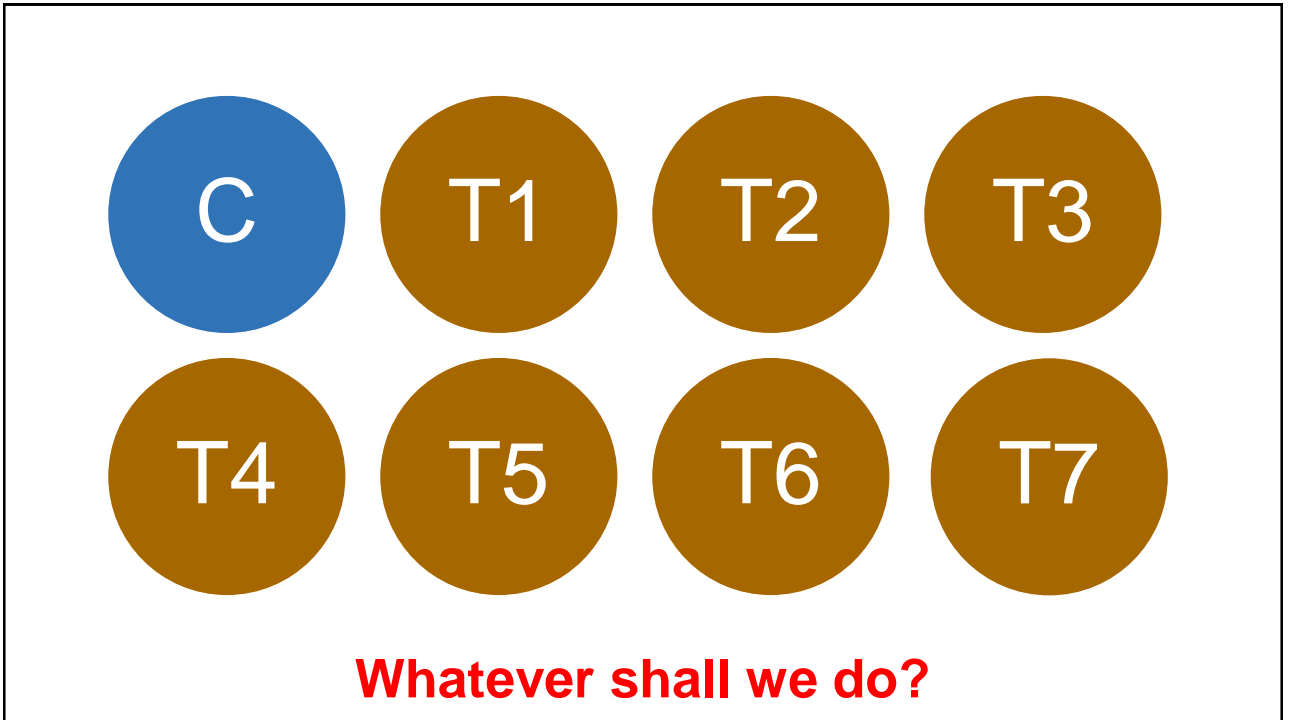
Bair, Hastie &amp; Tibshirani (2006)

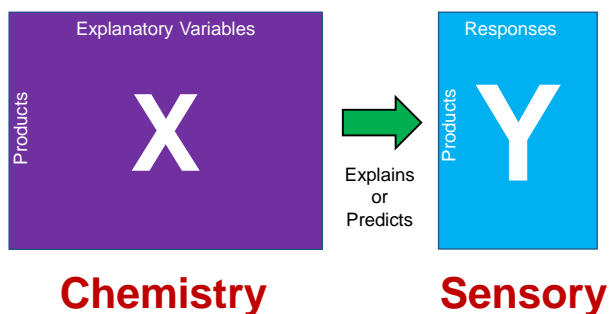


**Suppose there are many response variables**



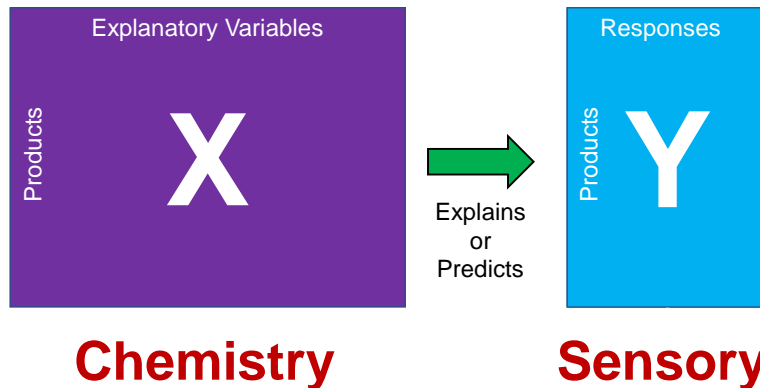






# Partial least squares regression

## Explaining and predicting data relationships



## Partial least squares regression (PLSR)

We want to ***explain*** and ***predict*** multivariate **Y** from the multivariate **X**.

Successive PLS components extract ***covariation*** between **X** and **Y** maximally.

## Partial least squares regression (PLSR)

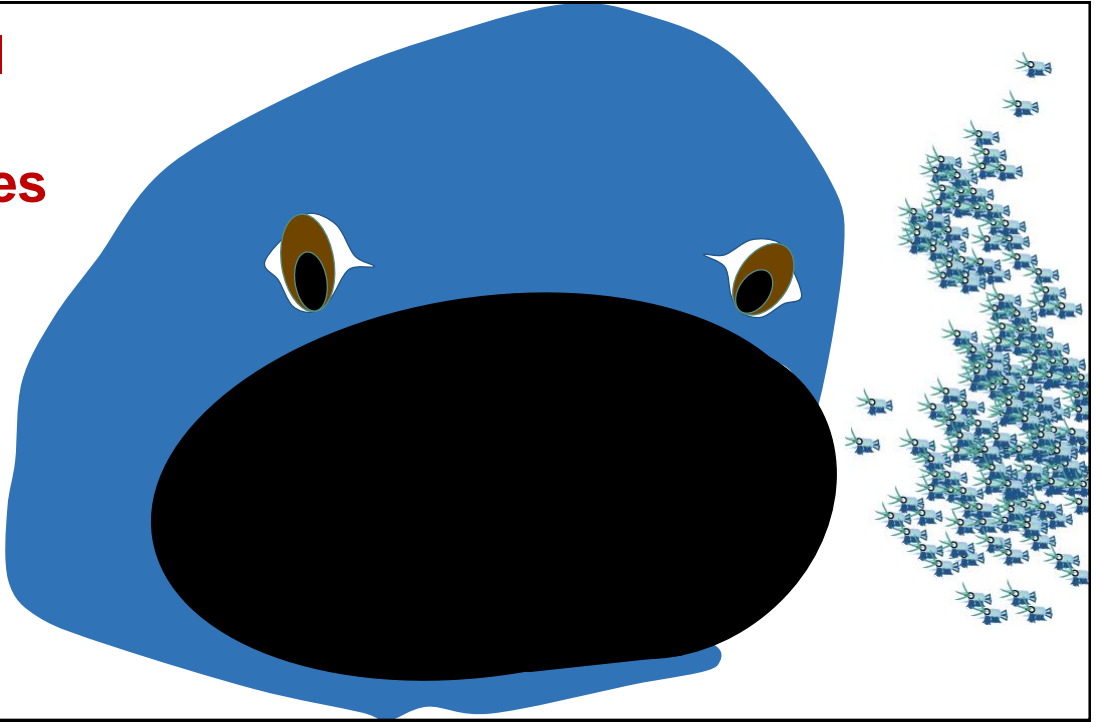
We want to ***explain*** and ***predict*** multivariate **Y** from the multivariate **X**.

Successive **PLS components** extract ***covariation*** between **X** and **Y** maximally.

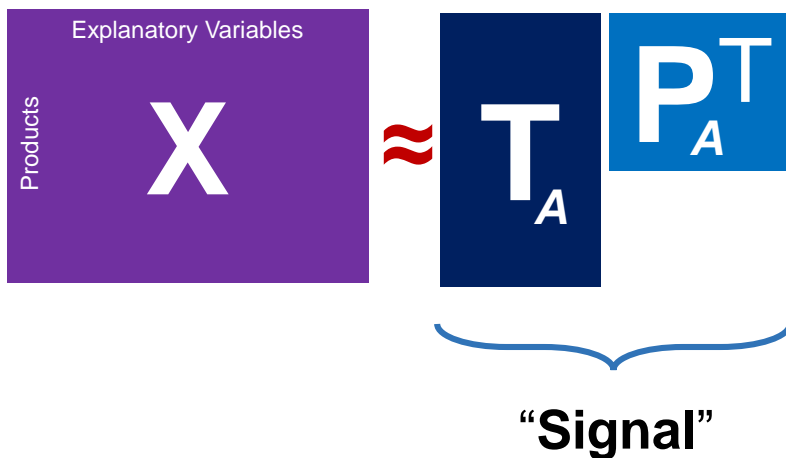
## PLS component



## Partial least squares



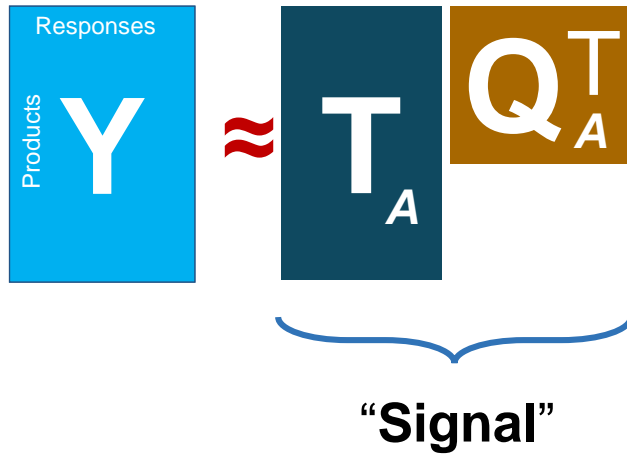
## Explaining and predicting data relationships



Wold, H. (1966). Estimation of principal components and related models by iterative least squares. In Krishnaiah, P.R. (ed.), *Multivariate Analysis*.

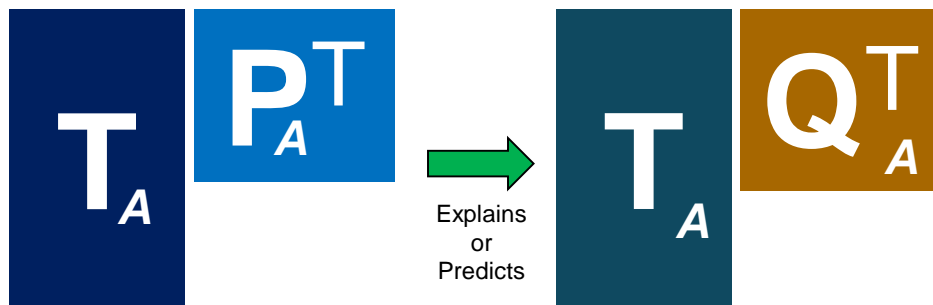


## Explaining and predicting data relationships



Wold (1966)

## Partial least squares regression (PLSR)



Wold (1966)

## Explaining and predicting data relationships

$$X \cdot B = Y$$

PLS regression coefficients

Wold (1966)

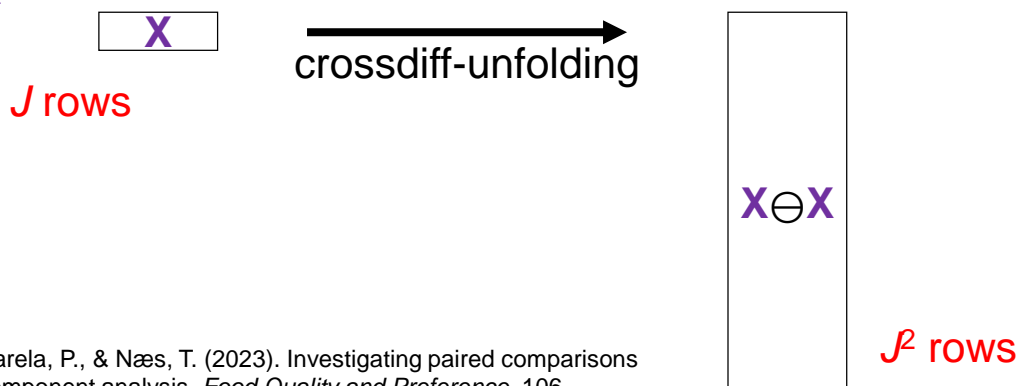




# Investigating paired comparisons

## Crossdiff-unfolding\*

Subtract every row in  $X$  from every row in  $X$



\* See

Castura, J.C., Varela, P., & Næs, T. (2023). Investigating paired comparisons after principal component analysis. *Food Quality and Preference*, 106, 104814. <https://doi.org/10.1016/j.foodqual.2023.104814>

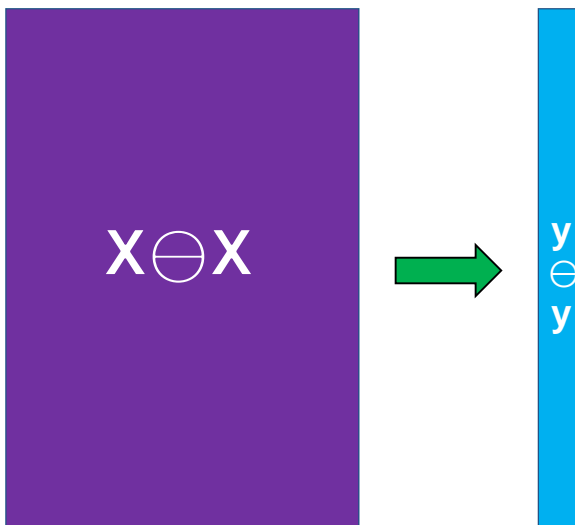
## (S)PCR of all paired comparisons

Goal is to ***explain*** and ***predict*** all response paired comparisons from all explanatory paired comparisons.

Regress  $y \ominus y$  on  $X \ominus X$ .

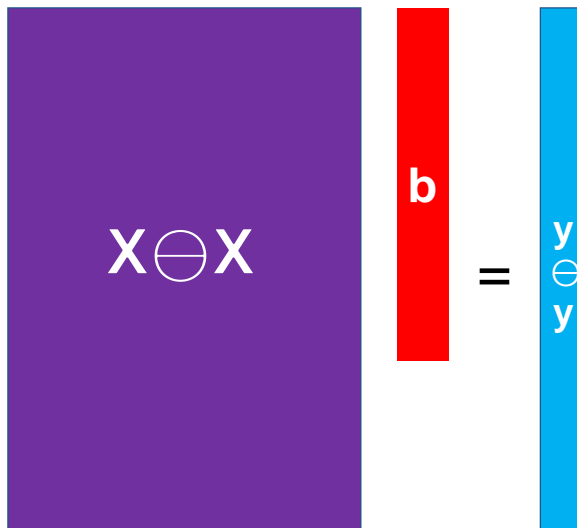
Castura & Tomic (2024)

## (S)PCR of all paired comparisons



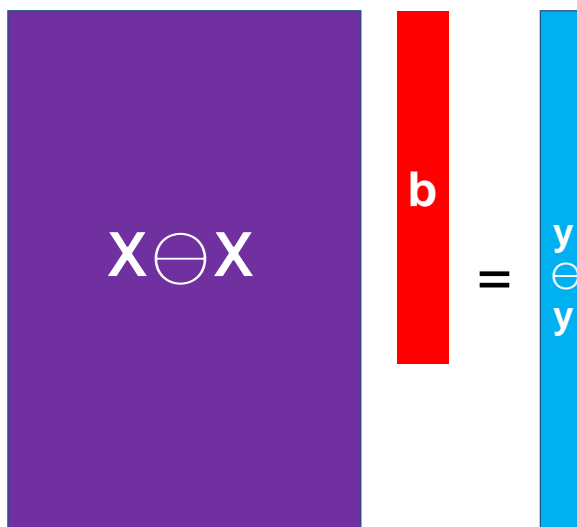
Castura & Tomic (2024)

## (S)PCR of all paired comparisons



Castura &amp; Tomic (2024)

## (S)PCR of all paired comparisons



Castura &amp; Tomic (2024)



## PLSR of all paired comparisons

 $X \ominus X$ 

 $Y \ominus Y$ 

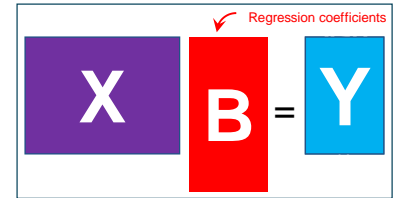
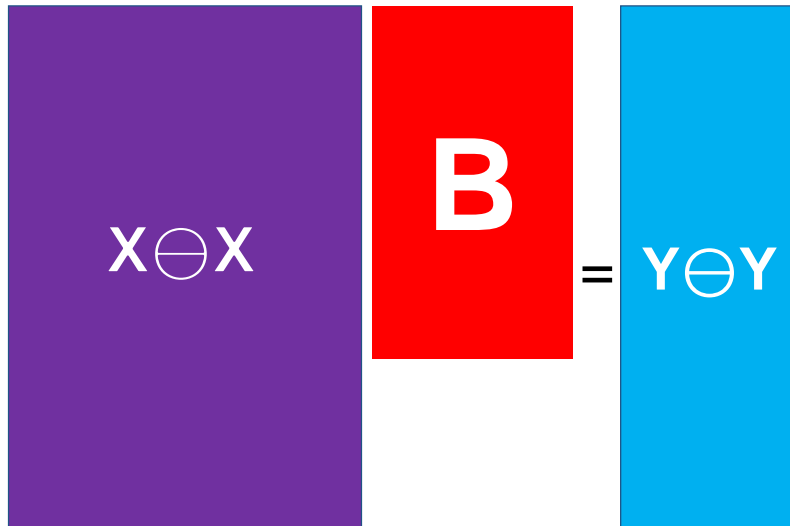
Castura, Tomic & Næs (2024)

## PLSR of all paired comparisons

 $X \ominus X$ 
 $B$ 
 $=$ 
 $Y \ominus Y$ 

Castura, Tomic & Næs (2024)

## PLSR of all paired comparisons



Castura, Tomic & Næss (2024)

## A subset of paired comparisons



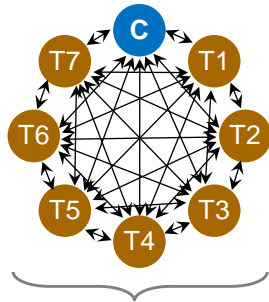
If a subset of paired comparisons is of primary interest, then...

we want to focus on this subset which contains the *relevant* variation

Castura & Tomic (2024)

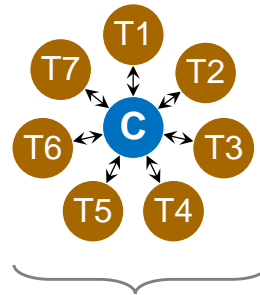
## All pairs vs. a subset of paired comparisons

### All Pairs



28 paired comparisons  
56 paired differences

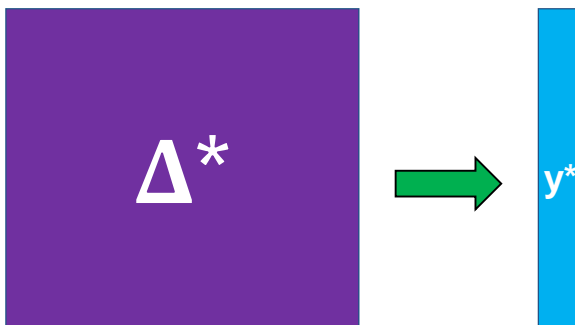
### Test-Control Pairs



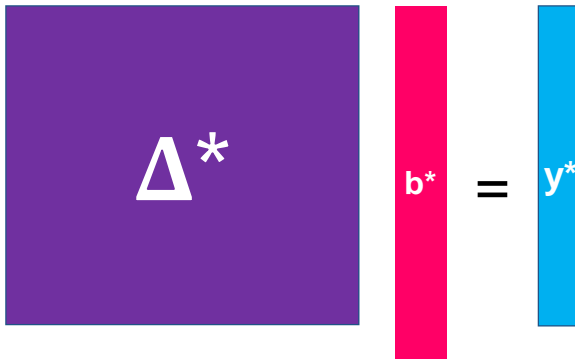
7 paired comparisons  
14 paired differences

Castura, J.C., Varela, P., & Næs, T. (2023). Investigating only a subset of paired comparisons after principal component analysis. *Food Quality and Preference*, 110, 104941. <https://doi.org/10.1016/j.foodqual.2023.104941>

## (S)PCR of a subset of paired comparisons

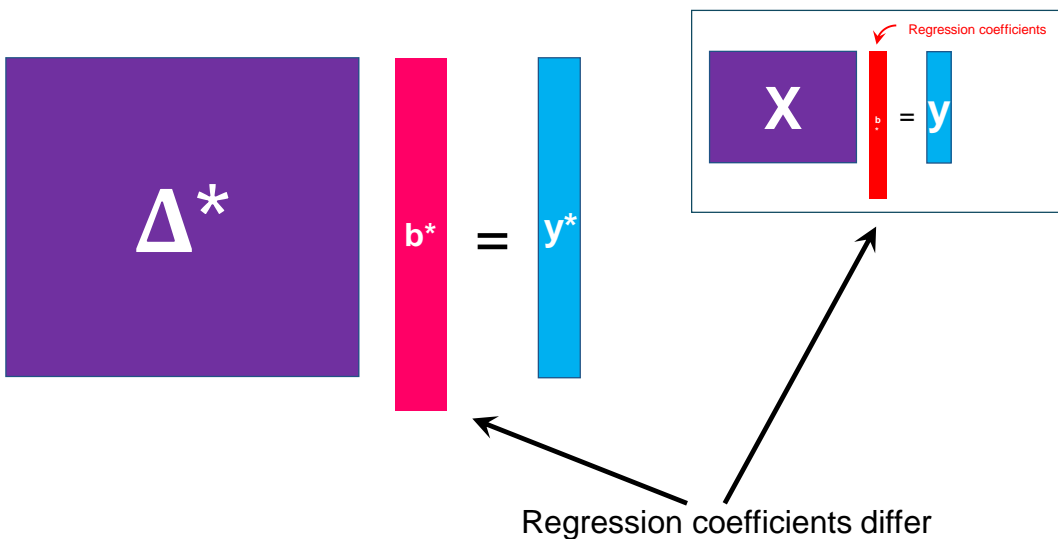


## (S)PCR of a subset of paired comparisons



Castura &amp; Tomic (2024)

## (S)PCR of a subset of paired comparisons



Castura &amp; Tomic (2024)



## (S)PCR of a subset of paired comparisons



SPCR of a subset of paired comparisons always explains and often predicts these paired comparisons better than conventional PCR

Regression coefficients differ

$$B = y$$

Regression coefficients

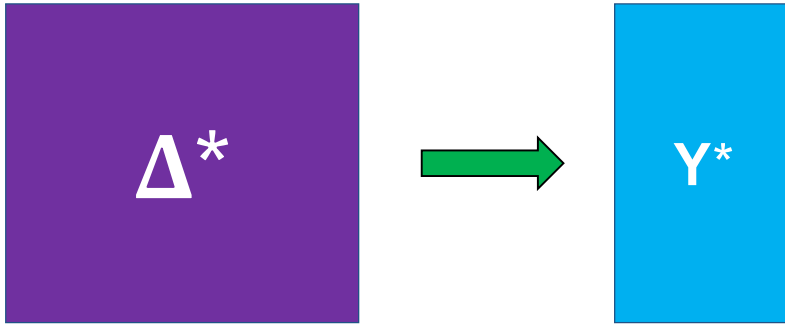
Castura & Tomic (2024)

## SPCR of a subset of paired comparisons

Focus on relevant variables / columns

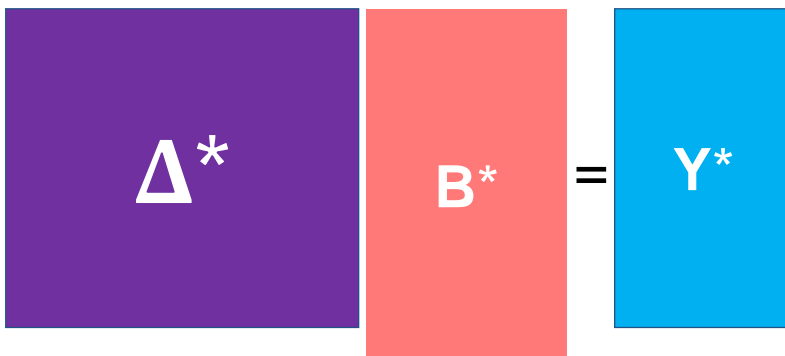
Focus on relevant paired comparisons / rows

## PLSR of a subset of paired comparisons



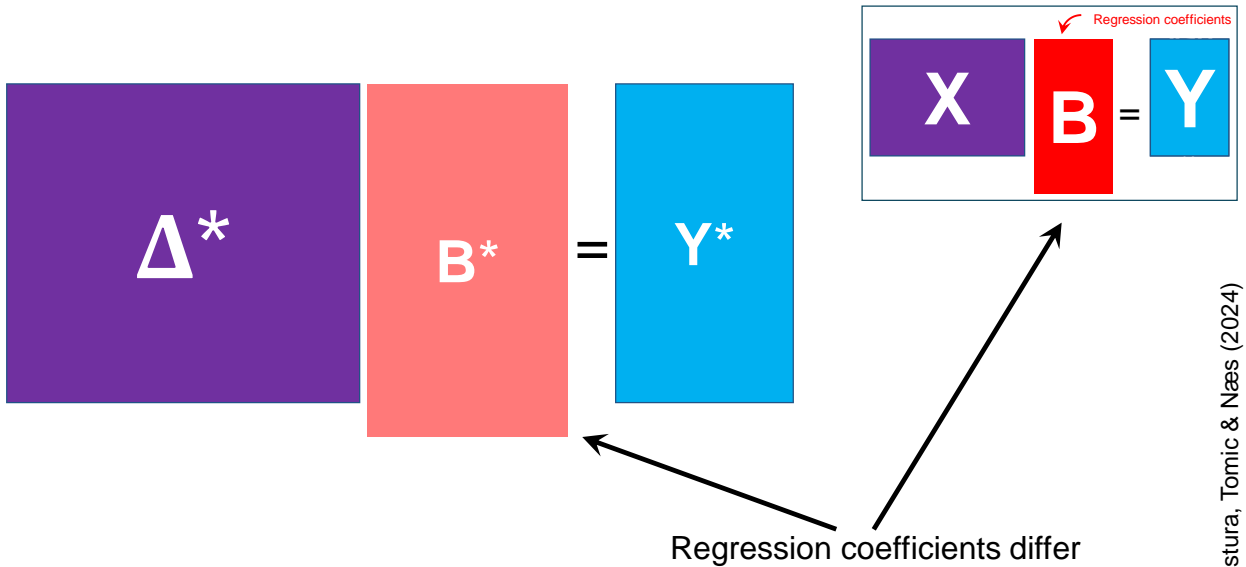
Castura, Tomic &amp; Næs (2024)

## PLSR of a subset of paired comparisons

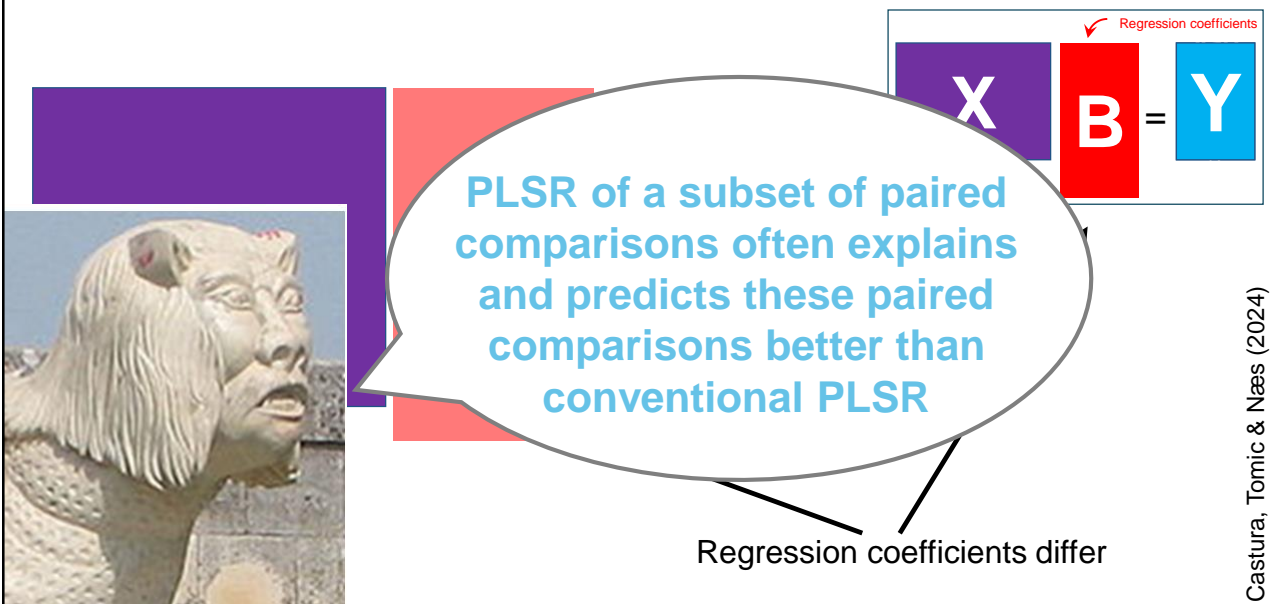


Castura, Tomic &amp; Næs (2024)

## PLSR of a subset of paired comparisons



## PLSR of a subset of paired comparisons





## References from 2024

- Castura, J.C., & Tomic, O. (2024). Supervised principal component regression of select paired comparisons. *Manuscript under review. To be presented at 17th International Weurman Flavour Research Symposium. 24-27 September 2024. Wageningen University & Research, Wageningen, The Netherlands.*
- Castura, J.C., Tomic, O., & Næs, T. (2024). Partial least squares regression of select paired comparisons. *16th AgroStat Conference. 3-6 September 2024. Bragança, Portugal.*
- Castura, J.C., Cariou, V., & Næs, T. (2024). Investigating control-centred results after uncentred principal component analysis. *Zenodo (preprint).* <https://doi.org/10.5281/zenodo.11496201>

## References from 2023

- Castura, J.C., Varela, P., & Næs, T. (2023). Investigating paired comparisons after principal component analysis. *Food Quality and Preference*, 106, 104814. <https://doi.org/10.1016/j.foodqual.2023.104814>
- Castura, J.C., Varela, P., & Næs, T. (2023) Evaluation of complementary numerical and visual approaches for investigating pairwise comparisons after principal component analysis. *Food Quality and Preference*, 107, 104843. <https://doi.org/10.1016/j.foodqual.2023.104843>
- Castura, J.C., Varela, P., & Næs, T. (2023). Investigating only a subset of paired comparisons after principal component analysis. *Food Quality and Preference*, 110, 104941. <https://doi.org/10.1016/j.foodqual.2023.104941>
- Næs, T., Varela, P., Castura, J.C., Bro, R., & Tomic, O. (2023). Why use component-based methods in sensory science? *Food Quality and Preference*, 112, 105028. <https://doi.org/10.1016/j.foodqual.2023.105028>





John Castura



## Acknowledgements...



Oliver Tomic



Paula Varela



Tormod Næs



For further information, please contact [jcastura@compusense.com](mailto:jcastura@compusense.com)

